

# INTERNATIONAL STANDARD

# ISO 3951-1

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## Sampling procedures for inspection by variables —

### Part 1: Specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection for a single quality characteristic and a single AQL

*Règles d'échantillonnage pour les contrôles par mesures —*

*Partie 1: Spécification pour les plans d'échantillonnage simples  
indexés d'après une limite de qualité acceptable (LQA) pour un  
contrôle lot par lot pour une caractéristique-qualité unique et une  
LQA unique*



Reference number  
ISO 3951-1:2013(E)

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## ISO 3951-1:2013(E)



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## ISO 3951-1:2013(E)

### Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see [www.iso.org/directives](http://www.iso.org/directives)).

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights. Details of any patent rights identified during the development of the document will be in the Introduction and/or on the ISO list of patent declarations received (see [www.iso.org/patents](http://www.iso.org/patents)).

Any trade name used in this document is information given for the convenience of users and does not constitute an endorsement.

For an explanation on the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the WTO principles in the Technical Barriers to Trade (TBT) see the following URL: [Foreword - Supplementary information](#)

The committee responsible for this document is ISO/TC 69, *Application of statistical methods*, SC 5, *Acceptance sampling*.

This second edition cancels and replaces the first edition (ISO 3951-1:2006), of which it constitutes a minor revision with the following changes:

- procedures have been introduced to accommodate measurement uncertainty;
- many of the sampling plans have been adjusted to improve the match between their operating characteristic curves and the operating characteristic curves of the corresponding plans for single sampling by attributes in ISO 2859-1.

ISO 3951 consists of the following parts, under the general title *Sampling procedures for inspection by variables*:

- *Part 1: Specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection for a single quality characteristic and a single AQL*
- *Part 2: General specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection of independent quality characteristics*
- *Part 3: Double sampling schemes indexed by acceptance quality limit (AQL) for lot-by-lot inspection*
- *Part 4: Procedures for assessment of declared quality levels*
- *Part 5: Sequential sampling plans indexed by acceptance quality limit (AQL) for inspection by variables (known standard deviation)*

This corrected version of ISO 3951-1:2013 incorporates the following corrections:

- [Clause 4](#), process mean (6<sup>th</sup> line): the symbol “*m*” has been replaced with “ $\mu$ ”, and “process mean” has been replaced with “unknown process mean”;
- [16.2](#), before Example 1: “ $Q_U \leq k$ ” and “ $Q_L \leq k$ ” have been replaced with “ $Q_U < k$ ” and “ $Q_L < k$ ”;

- [16.4.2, Formula \(4\)](#): “ $\left[ \sqrt{(1 - Q\sqrt{3} / 2) / 2} \right]$ ” has been replaced with “ $\left[ \sqrt{(1 - Q\sqrt{3} / 2) / 2} \right]$ ”;
- [17.2](#), Example: “it is seen that for an AQL of 1,0 %” has been replaced with “it is seen that for an AQL of 0,65 %”;
- [N.2, Formula \(N.2\)](#): “ $\left[ (1 - Q\sqrt{3} / 2) 2 \right]$ ” has been replaced with “ $\left[ (1 - Q\sqrt{3} / 2) / 2 \right]$ ”;
- [O.4, Formula \(O.6\)](#): in the fourth line, the question mark has been replaced with a “0”, thus correcting a typographic error.

## ISO 3951-1:2013(E)

### Introduction

This part of ISO 3951 specifies an acceptance sampling system of single sampling plans for inspection by variables. It is indexed in terms of the acceptance quality limit (AQL) and is designed for users who have simple requirements. (A more comprehensive and technical treatment is given in ISO 3951-2.) This part of ISO 3951 is complementary to ISO 2859-1.

The objectives of the methods laid down in this part of ISO 3951 are to ensure that lots of acceptable quality have a high probability of acceptance and that the probability of not accepting inferior lots is as high as practicable. This is achieved by means of the switching rules, which provide the following:

- a) an automatic protection to the consumer (by means of a switch to tightened inspection or discontinuation of sampling inspection) should a deterioration in quality be detected;
- b) an incentive (at the discretion of the responsible authority) to reduce inspection costs (by means of a switch to a smaller sample size) should consistently good quality be achieved.

In this part of ISO 3951, the acceptability of a lot is implicitly determined from an estimate of the percentage of nonconforming items in the process, based on a random sample of items from the lot.

This part of ISO 3951 is intended for application to a continuing series of lots of discrete products all supplied by one producer using one production process. If there are different producers or production processes, this part of ISO 3951 is applied to each one separately.

This part of ISO 3951 is intended for application to a single quality characteristic that is measurable on a continuous scale. For two or more such quality characteristics, see ISO 3951-2.

It is assumed in the body of this part of ISO 3951 that measurement error is negligible (see ISO 10576-1:2003). For information on allowing for measurement error, see Annex O, which was derived from Reference [20] in the Bibliography.

For double specification limits, this part of ISO 3951 treats combined control. For other types of control, refer to ISO 3951-2.

**CAUTION — The procedures in this part of ISO 3951 are not suitable for application to lots that have been screened for nonconforming items.**

Inspection by variables for percent nonconforming items, as described in this part of ISO 3951, includes several possible modes, the combination of which leads to a presentation that may appear quite complex to the user:

- unknown standard deviation, or originally unknown then estimated with fair precision, or known since the start of inspection;
- a single specification limit, or combined control of double specification limits;
- normal inspection, tightened inspection, or reduced inspection.

[Table 1](#) is intended to facilitate the use this part of ISO 3951 by directing the user to the paragraphs and tables concerning any situation with which he may be confronted. The table only deals with [Clauses 15](#), [16](#), [20](#), [21](#), and [22](#); in every case, it is necessary, first of all, to have read the other clauses.



Table 1 — Summary table

Inspection	Single specification limit						Double specification limits with combined control					
	s-method			σ-method			s-method			σ-method		
	Clauses or sub-clauses	Tables/ Annexes	Charts	Clauses or sub-clauses	Tables/ Annexes	Charts	Clauses or sub-clauses	Tables/ Annexes	Charts	Clauses or sub-clauses	Tables/ Annexes	Charts
Normal inspection	<a href="#">16.1, 16.2, 16.3, 21.1</a>	<a href="#">A.1, B.1</a> , B to R	B to R	<a href="#">17.1, 17.2, 21.1</a>	<a href="#">A.1, C.1</a> , B to R <sup>a</sup>	B to R <sup>a</sup>	<a href="#">16.1, 16.4, 21.1</a>	<a href="#">A.1, D.1, E.1</a> (for $n = 3$ ), <a href="#">G.1</a> (for $n = 3$ or 4), B to R <sup>a</sup>	s-D to s-R, B to R <sup>a</sup>	<a href="#">17.1, 17.3</a> and <a href="#">21.1</a>	<a href="#">A.1, C.1, E.1</a> , B to R <sup>a</sup>	B to R <sup>a</sup>
Switching between normal and tightened inspection	<a href="#">21.2, 21.3</a>	<a href="#">B.1, B.2</a>	B to R	<a href="#">21.2, 21.3</a>	<a href="#">C.1, C.2</a>	B to R <sup>a</sup>	<a href="#">21.2, 21.3</a>	<a href="#">D.1, D.2</a>	s-D to s-R, B to R <sup>a</sup>	<a href="#">21.2, 21.3</a>	<a href="#">C.1, C.2, E.1</a>	B to R <sup>a</sup>
Switching between normal and reduced inspection	<a href="#">21.4, 21.5</a>	<a href="#">B.1, B.3</a>	B to R	<a href="#">21.4, 21.5</a>	<a href="#">C.1, C.3, I</a>	B to R <sup>a</sup>	<a href="#">21.4, 21.5</a>	<a href="#">D.1, D.3, G.1</a> (for $n = 3$ or 4)	s-D to s-R, B to R <sup>a</sup>	<a href="#">21.4, 21.5</a>	<a href="#">C.1, C.3, E.1</a>	B to R <sup>a</sup>
Switching between tightened and discontinued inspection	<a href="#">22</a>	<a href="#">B.2</a>	B to R	<a href="#">22</a>	<a href="#">C.2</a>	B to R <sup>a</sup>	<a href="#">22</a>	<a href="#">D.2</a>	s-D to s-R, B to R <sup>a</sup>	<a href="#">22</a>	<a href="#">E.1</a>	B to R <sup>a</sup>
Switching between the s-method and σ-method	<a href="#">23</a>	Annex J		<a href="#">23</a>	Annex J		<a href="#">23</a>	Annex J		<a href="#">23</a>	Annex E, Annex J	

<sup>a</sup> But see [8.4](#).

Fifteen annexes are provided. Annexes A to I provide the tables needed to support the procedures. Annex J indicates how the sample standard deviation,  $s$ , and the presumed known value of the process standard deviation,  $\sigma$ , should be determined. Annex K provides the statistical theory underlying the calculation of the consumer's risk qualities, together with tables showing these quality levels for normal, tightened, and reduced inspection as well as for the  $s$ -method and  $\sigma$ -method. Annex L provides similar information for the producer's risks. Annex M gives the general formula for the operating characteristic of the  $\sigma$ -method. Annex N provides the statistical theory underlying the estimation of the process fraction nonconforming under the  $s$ -method for sample sizes 3 and 4, which, for technical reasons, are treated differently from the other sample sizes in this part of ISO 3951. Annex O provides procedures for accommodating measurement uncertainty.



# Sampling procedures for inspection by variables —

## Part 1:

# Specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection for a single quality characteristic and a single AQL

## 1 Scope

This part of ISO 3951 is primarily designed for use under the following conditions:

- a) where the inspection procedure is to be applied to a continuing series of lots of discrete products all supplied by one producer using one production process;
- b) where only a single quality characteristic,  $x$ , of these products is taken into consideration, which must be measurable on a continuous scale;
- c) where production is stable (under statistical control) and the quality characteristic,  $x$ , is distributed according to a normal distribution or a close approximation to the normal distribution;
- d) where a contract or standard defines a lower specification limit,  $L$ , an upper specification limit,  $U$ , or both; an item is qualified as conforming if and only if its measured quality characteristic,  $x$ , satisfies the appropriate one of the following inequalities:
  - 1)  $x \geq L$  (i.e. the lower specification limit is not violated);
  - 2)  $x \leq U$  (i.e. the upper specification limit is not violated);
  - 3)  $x \geq L$  and  $x \leq U$  (i.e. neither the lower nor the upper specification limit is violated).

Inequalities 1) and 2) are called cases with a single specification limit and 3), a case with double specification limits.

Where double specification limits apply, it is assumed in this part of ISO 3951 that conformance to both specification limits is equally important to the integrity of the product. In such cases, it is appropriate to apply a single AQL to the combined percentage of a product outside the two specification limits. This is referred to as combined control.

## 2 Normative references

The following documents, in whole or in part, are normatively referenced in this document and are indispensable for its application. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 2859-1, *Sampling procedures for inspection by attributes — Part 1: Sampling schemes indexed by acceptance quality limit (AQL) for lot-by-lot inspection*

ISO 2859-2, *Sampling procedures for inspection by attributes — Part 2: Sampling plans indexed by limiting quality (LQ) for isolated lot inspection*

ISO 3534-1, *Statistics — Vocabulary and symbols — Part 1: General statistical terms and terms used in probability*

ISO 3534-2, *Statistics — Vocabulary and symbols — Part 2: Applied statistics*

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ISO 3951-2, *Sampling procedures for inspection by variables — Part 2: General specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection of independent quality characteristics*

### 3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 2859-1, ISO 3534-1, and ISO 3534-2 and the following apply.

#### 3.1

##### **inspection by variables**

inspection by measuring the magnitude of a characteristic of an item

[SOURCE: ISO 3534-2]

#### 3.2

##### **sampling inspection**

inspection of selected items in the group under consideration

[SOURCE: ISO 3534-2]

#### 3.3

##### **acceptance sampling inspection**

##### **acceptance sampling**

*sampling inspection* (3.2) to determine whether or not to accept a lot or other amount of product, material, or service

[SOURCE: ISO 3534-2]

#### 3.4

##### **acceptance sampling inspection by variables**

*acceptance sampling inspection* (3.3) in which the acceptability of the process is determined statistically from measurements on specified quality characteristics of each item in a sample from a lot

#### 3.5

##### **process fraction nonconforming**

rate at which nonconforming items are generated by a process

Note 1 to entry: It is expressed as a proportion.

#### 3.6

##### **acceptance quality limit**

##### **AQL**

worst tolerable *process fraction nonconforming* (3.5) when a continuing series of lots is submitted for *acceptance sampling* (3.3)

Note 1 to entry: See [Clause 5](#).

#### 3.7

##### **quality level**

quality expressed as a rate of occurrence of nonconforming units

#### 3.8

##### **limiting quality**

##### **LQ**

*quality level* (3.7), when a lot is considered in isolation, which, for the purposes of *acceptance sampling inspection* (3.3), is limited to a low probability of acceptance

[SOURCE: ISO 3534-2]

Note 1 to entry: See [14.1](#).

Note 2 to entry: In this part of ISO 3951: 10 %.

### 3.9

#### **nonconformity**

non-fulfilment of a requirement

### 3.10

#### **nonconforming unit**

unit with one or more nonconformities

[SOURCE: ISO 3534-2]

### 3.11

#### **s-method acceptance sampling plan**

*acceptance sampling* (3.3) plan by variables using the sample standard deviation

[SOURCE: ISO 3534-2]

Note 1 to entry: See [Clause 16](#).

### 3.12

#### **$\sigma$ -method acceptance sampling plan**

*acceptance sampling* (3.3) plan by variables using the presumed value of the process standard deviation

[SOURCE: ISO 3534-2]

Note 1 to entry: See [Clause 17](#).

### 3.13

#### **specification limit**

conformance boundary specified for a characteristic

[SOURCE: ISO 3534-2]

### 3.14

#### **lower specification limit**

*L*

*specification limit* (3.13) that defines the lower conformance boundary

[SOURCE: ISO 3534-2]

### 3.15

#### **upper specification limit**

*U*

*specification limit* (3.13) that defines the upper conformance boundary

[SOURCE: ISO 3534-2]

### 3.16

#### **combined control**

requirement when both upper and lower limits are specified for the quality characteristic and an *AQL* (3.6) that applies to the combined percent nonconforming beyond the two limits is given

Note 1 to entry: See [5.3](#).

Note 2 to entry: The use of combined control implies that nonconformity beyond either *specification limit* (3.13) is believed to be of equal, or at least roughly equal, importance to the lack of integrity of the product.

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### 3.17 acceptability constant

$k$

constant depending on the specified value of the *acceptance quality limit* (3.6) and the sample size, used in the criteria for accepting the lot in an *acceptance sampling* (3.3) plan by variables

[SOURCE: ISO 3534-2]

Note 1 to entry: See [16.2](#) and [17.2](#).

### 3.18 quality statistic

$Q$

function of the *specification limit* (3.13), the sample mean, and the sample or process standard deviation used in assessing the acceptability of a lot

[SOURCE: ISO 3534-2]

Note 1 to entry: For the case of a single *specification limit* (3.13), the lot may be sentenced on the result of comparing  $Q$  with the *acceptability constant* (3.17)  $k$ .

Note 2 to entry: See [16.2](#) and [17.2](#).

### 3.19 lower quality statistic

$Q_L$

function of the *lower specification limit* (3.14), the sample mean, and the sample or process standard deviation

Note 1 to entry: For a single *lower specification limit* (3.14), the lot is sentenced on the result of comparing  $Q_L$  with the *acceptability constant* (3.17)  $k$ .

[SOURCE: ISO 3534-2]

Note 2 to entry: See [Clause 4](#), [16.2](#), and [17.2](#).

### 3.20 upper quality statistic

$Q_U$

function of the *upper specification limit* (3.15), the sample mean, and the sample or process standard deviation

Note 1 to entry: For a single *upper specification limit* (3.15), the lot is sentenced on the result of comparing  $Q_U$  with the *acceptability constant* (3.17)  $k$ .

[SOURCE: ISO 3534-2]

Note 2 to entry: See [Clause 4](#), [16.2](#), and [17.2](#).

### 3.21 maximum sample standard deviation MSSD

$S_{\max}$

largest sample standard deviation for a given sample size code letter, inspection severity, and *acceptance quality limit* (3.6) for which it is possible to satisfy the acceptance criteria for the combined control of double *specification limits* (3.13) when the process variability is unknown

Note 1 to entry: See [16.4](#).

### 3.22 maximum process standard deviation MPSD

$\sigma_{\max}$

largest process standard deviation for a given sample size code letter and *acceptance quality limit* (3.6) for which it is possible to satisfy the acceptance criterion for double specification limits with a combined *AQL* (3.6) requirement under tightened inspection with known process variability

[SOURCE: ISO 3534-2]

Note 1 to entry: See [17.3](#).

### 3.23 switching rule

instruction within an *acceptance sampling* (3.3) scheme for changing from one *acceptance sampling* (3.3) plan to another of greater or lesser severity based on demonstrated quality history

[SOURCE: ISO 3534-2]

Note 1 to entry: Normal, tightened, or reduced inspection or discontinuation of inspection are examples of 'severity'.

Note 2 to entry: See [Clause 21](#).

### 3.24 measurement

set of operations to determine the value of some quantity

[SOURCE: ISO 3534-2]

## 4 Symbols

The symbols used are as follows:

$c_U$	factor for determining the upper control limit for the sample standard deviation (See Annex H.)
$f_s$	factor that relates the maximum sample standard deviation to the difference between $U$ and $L$ (See Annex D)
$f_\sigma$	factor that relates the maximum process standard deviation under tightened inspection to the difference between $U$ and $L$ (See Annex E)
$k$	Form $k$ acceptability constant for use with a single quality characteristic and a single specification limit (See Annex B for the $s$ -method or Annex C for the $\sigma$ -method)
$L$	lower specification limit (As a subscript to a variable, it denotes its value at $L$ .)
$\mu$	unknown process mean
$N$	lot size (number of items in a lot)
$n$	sample size (number of items in a sample)
$\hat{p}$	estimate of the process fraction nonconforming
$\hat{p}_L$	estimate of the process fraction nonconforming below the lower specification limit
$\hat{p}_U$	estimate of the process fraction nonconforming above the upper specification limit
$p^*$	maximum acceptable value for the estimate of the process fraction nonconforming
$P_a$	probability of acceptance
$Q$	quality statistic
$Q_L$	lower quality statistic

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NOTE  $Q_L$  is defined as  $(\bar{x} - L)/s$  when the process standard deviation is unknown, and as  $(\bar{x} - L)/\sigma$  when it is presumed to be known.

$Q_U$  upper quality statistic

NOTE  $Q_U$  is defined as  $(U - \bar{x})/s$  when the process standard deviation is unknown, and as  $(U - \bar{x})/\sigma$  when it is presumed to be known.

$s$  sample standard deviation of the measured values of the quality characteristic (also an estimate of the standard deviation of the process), i.e.

$$s = \sqrt{\frac{\sum_{j=1}^n (x_j - \bar{x})^2}{n-1}}$$

(See Annex J.)

$s_{\max}$  maximum sample standard deviation (MSSD)

$\sigma$  standard deviation of a process that is under statistical control

NOTE  $\sigma^2$ , the square of the process standard deviation, is known as the process variance.

$\sigma_{\max}$  maximum process standard deviation (MPSD)

$U$  upper specification limit (As a suffix to a variable, it denotes its value at  $U$ .)

$x_j$  measured value of the quality characteristic for the  $j^{\text{th}}$  item of the sample

$\bar{x}$  the arithmetic mean of the measured values of the quality characteristic in the sample, i.e.

$$\bar{x} = \frac{\sum_{j=1}^n x_j}{n}$$

## 5 Acceptance quality limit (AQL)

### 5.1 Concept

The AQL is the quality level that is the worst tolerable process fraction nonconforming when a continuing series of lots is submitted for acceptance sampling. Although individual lots with quality as bad as the AQL may be accepted with fairly high probability, the designation of an AQL does not suggest that this is a desirable quality level. The sampling schemes found in this part of ISO 3951, with their rules for switching and for discontinuation of sampling inspection, are designed to encourage suppliers to keep the process fractions nonconforming consistently better than the respective AQLs. Otherwise, there is a high risk that the inspection severity will be switched to tightened inspection, under which the criteria for lot acceptance become more demanding. Once on tightened inspection, unless action is taken to improve the process, it is very likely that the rule requiring discontinuation of sampling inspection will be invoked pending such improvement.

### 5.2 Use

The AQL, together with the sample size code letter, is used to index the sampling plans in this part of ISO 3951.

### 5.3 Specifying AQLs

The AQL to be used will be designated in the product specification or in the contract, or by the responsible authority. Where both upper and lower specification limits are given, this part of ISO 3951 addresses only the case of an overall AQL applying to the combined percent nonconforming beyond the



two limits; this is known as “combined control.” (See ISO 3951-2 for “separate” and “complex” control of double specification limits.)

## 5.4 Preferred AQLs

The 16 AQLs given in this part of ISO 3951, ranging in value from 0,01 % to 10 % nonconforming, are described as preferred AQLs. They are only preferred in the sense that they are the AQL values used in the tabulations and charts. It follows that, if for any product or service, an AQL other than a preferred AQL is designated, then this part of ISO 3951 is not applicable. (See [14.2](#).)

## 5.5 Caution

From the definition of the AQL in [5.1](#), it follows that the desired protection can only be ensured when a continuing series of lots is provided for inspection.

## 5.6 Limitation

The designation of an AQL shall not imply that the supplier has the right to supply knowingly any nonconforming product.

# 6 Switching rules for normal, tightened, and reduced inspection

Switching rules discourage the producer from operating at a quality level that is worse than the AQL. This part of ISO 3951 prescribes a switch to tightened inspection when inspection results indicate that the AQL is being exceeded. It further prescribes a discontinuation of sampling inspection altogether if tightened inspection fails to stimulate the producer into rapidly improving his production process.

Tightened inspection and the discontinuation rule are integral and, therefore, obligatory procedures of this part of ISO 3951 if the protection implied by the AQL is to be maintained.

This part of ISO 3951 also provides the possibility of switching to reduced inspection when inspection results indicate that the quality level is stable and reliable at a level better than the AQL. This practice is, however, optional (at the discretion of the responsible authority).

When there is sufficient evidence from the control charts (see [20.1](#)) that the variability is in statistical control, consideration should be given to switching to the  $\sigma$ -method. If this appears advantageous, the consistent value of  $s$  (the sample standard deviation) shall be taken as  $\sigma$  (see [Clause 23](#)).

When it has been necessary to discontinue acceptance sampling inspection, inspection under this part of ISO 3951 shall not be resumed until action has been taken by the producer to improve the quality of the submitted product.

Details of the operation of the switching rules are given in [Clauses 21, 22, and 23](#).

# 7 Relation to ISO 2859-1

## 7.1 Similarities

The similarities are as follows.

- a) This part of ISO 3951 is complementary to ISO 2859-1; the two documents share a common philosophy and, as far as possible, their procedures and vocabulary are the same.
- b) Both use the AQL to index the sampling plans, and the preferred values used in this part of ISO 3951 are identical with those given for percent nonconforming in ISO 2859-1 (i.e. from 0,01 % to 10 %).
- c) In both International Standards, lot size and inspection level (inspection level II in default of other instructions) determine a sample size code letter. General tables give the sample size to be taken

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and the acceptability criterion, indexed by the sample size code letter and the AQL. Separate tables are given for the  $s$ -method and  $\sigma$ -method, and for normal, tightened, and reduced inspection.

- d) The switching rules are essentially equivalent.

### 7.2 Differences

- a) **Determination of acceptability.** Acceptability for an ISO 2859-1 attributes sampling plan for percent nonconforming is determined by the number of nonconforming items found in the sample. Acceptability for a plan for inspection by variables is based on the distance of the estimated value of the process mean from the specification limit(s) in terms of the estimated or presumed value of the process standard deviation. In this part of ISO 3951, two methods are considered: the  $s$ -method, for use when the process standard deviation,  $s$ , is unknown, and the  $\sigma$ -method, for use when  $s$  is presumed to be known. In the case of a single specification limit, the acceptability may be calculated from a formula (see 16.2 and 17.2), but for the  $s$ -method, it is also easily established by a graphical method (see 16.3). In the case of combined control of double specification limits under the  $s$ -method, this part of ISO 3951 provides only for a graphical method of determining acceptability (see 16.4); for combined control of double specification limits under the  $\sigma$ -method, a numerical method is given.
- b) **Normality.** In ISO 2859-1, there is no requirement relating to the distribution of the characteristics. However, in this part of ISO 3951, it is necessary for the efficient operation of the plans that the measurements be distributed according to a normal distribution or a close approximation to a normal distribution.
- c) **Operating characteristic curves (OC curves).** The OC curves of the variables plans in this part of ISO 3951 are not identical to those of the corresponding attributes plans in ISO 2859-1. The curves for unknown process standard deviation have been matched by minimizing the area between the curves representing the squares of the OC values, a method that gives greater emphasis to the match at the top of the OC curves. In most cases, the resulting match between the OC curves is so close that for most practical purposes, the attributes and variables OC curves may be considered to be identical. The plans for known process standard deviation were derived by minimizing the area between the squared OC functions subject to keeping the same form  $p^*$  acceptability constant as for the corresponding case for unknown process standard deviation, i.e. only the sample size was open to choice, so the match was, in general, less perfect.
- d) **Producer's risk.** For process quality precisely at the AQL, the producer's risk that a lot will not be accepted tends to decrease with one-step increases in sample size coupled with one-step decreases in AQL, i.e. down diagonals of the master tables running from top right to bottom left. The progressions of probabilities are similar, but not identical, to those in ISO 2859-1.

NOTE The producer's risks of the plans are given in Annex L.

- e) **Sample sizes.** The variables sample sizes for given combinations of sample size code letter and AQL are usually smaller than the corresponding attributes sample sizes. This is particularly true for the  $\sigma$ -method. Moreover, due to the method by which the variables plans were derived, their sample sizes vary over AQL for a given sample size code letter.
- f) **Double sampling plans.** Double sampling plans by variables are presented separately, in ISO 3951-3.
- g) **Multiple sampling plans.** No multiple sampling plans by variables are given in this part of ISO 3951.
- h) **Average outgoing quality limit (AOQL).** The AOQL concept is mainly of value when 100 % inspection and rectification is feasible for non-accepted lots. It follows that the AOQL concept cannot be applied under destructive or expensive testing. As variables plans will generally be used under these circumstances, no tables of AOQL have been included in this part of ISO 3951.

## 8 Consumer protection

### 8.1 Use of individual plans

This part of ISO 3951 is intended to be used as a system employing tightened, normal, and reduced inspection on a continuing series of lots to provide consumer protection while assuring the producer that acceptance will be very likely to occur if quality is better than the AQL.

Some users may select specific individual plans from this part of ISO 3951 and use them without the switching rules. For example, a purchaser may be using the plans for verification purposes only. This is not the intended application of the system given in this part of ISO 3951 and its use in this way should not be referred to as “inspection in compliance with ISO 3951-1.” When used in such a way, ISO 3951-1 simply represents a collection of individual plans indexed by the AQL. The operating characteristic curves and other measures of a plan so chosen shall be assessed individually from the tables provided.

### 8.2 Consumer’s risk quality (CRQ) tables

If the series of lots is not long enough to allow the switching rules to be applied, it may be desirable to limit the selection of sampling plans to those, associated with a designated AQL value, that give a consumer’s risk quality (CRQ) not worse than the specified limiting quality protection. Sampling plans for this purpose can be selected by choosing a consumer’s risk quality and a consumer’s risk to be associated with it. Annex K gives values of consumer’s risk quality for the  $s$ -method and  $\sigma$ -method corresponding to a consumer’s risk of 10 %.

However, application of this part of ISO 3951 to isolated lots is deprecated, as the theory of sampling by variables applies to a process. For isolated lots, it is appropriate and more efficient to use plans for sampling by attributes, such as from ISO 2859-2. (See also Reference [5] in the Bibliography.)

### 8.3 Producer’s risk tables

Annex L gives the probability of non-acceptance under the  $s$ -method and  $\sigma$ -method for lots produced when the process fraction nonconforming equals the AQL. This probability is called the producer’s risk.

### 8.4 Operating characteristic (OC) curves

The tables for consumer’s risk quality and producer’s risk provide information about only two points on the operating characteristic curves. The degree of consumer protection provided by an individual sampling plan at any process quality level may, however, be judged from its operating characteristic curve. OC curves for the normal inspection  $s$ -method sampling plans of this part of ISO 3951 are given in Charts B to R, which should be consulted when choosing a sampling plan. Also given are tables of process qualities at nine standard probabilities of acceptance for all of the  $s$ -method sampling plans in this part of ISO 3951.

These OC curves and tables apply to a single specification limit under the  $s$ -method. Most of them also provide a good approximation to the  $\sigma$ -method and to the case of combined control of double specification limits, particularly for the larger sample sizes. If more accurate OC values are required for the  $\sigma$ -method, refer to Annex M.

## 9 Allowing for measurement uncertainty

The master tables of this part of ISO 3951 are based on the assumption that the quality characteristic,  $X$ , of the items in the lots is normally distributed with unknown process mean,  $\mu$ , and either known or unknown process standard deviation  $\sigma$ . The assumption is also made that  $X$  can be measured without measurement error, i.e. that measurement of an item with the true value,  $x_i$ , results in the value  $x_i$ . However, the master tables can also be used, with appropriate adjustments, in the presence of measurement error.